Wave Chaos for Electromagnetic Environment Modelling and Control

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Wave Modelling Group

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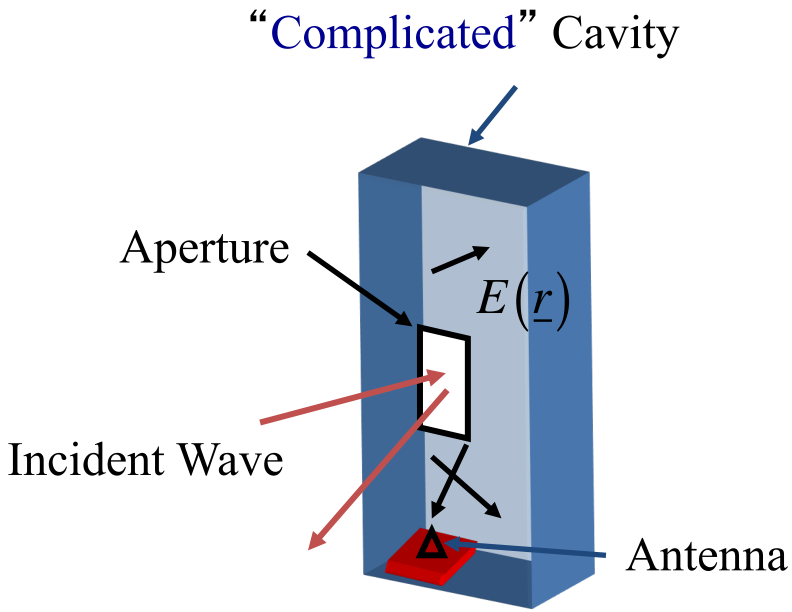
Modelling Problem

Problem: predict and control the propagation of waves in large and complicated electromagnetic environments.



1. Power enters cavity through aperture.
2. Distributes itself throughout.
3. Induces voltage on the terminals of an antenna.

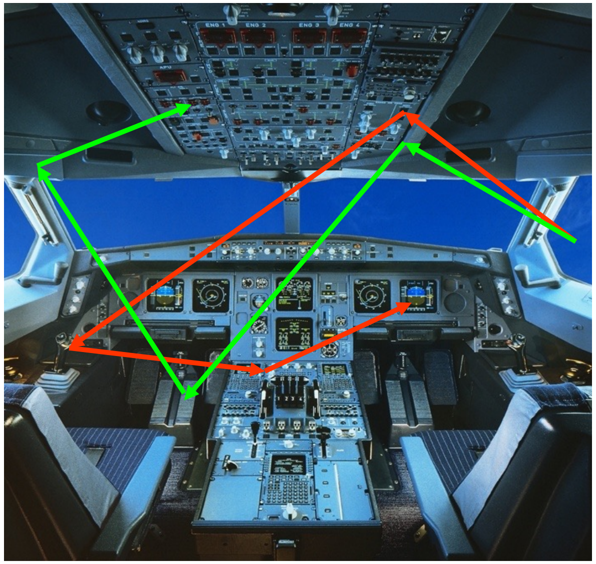
Complicated cavity model for:

- reverberation chambers.
- wireless propagation channels;
- avionic bay, naval compartments;

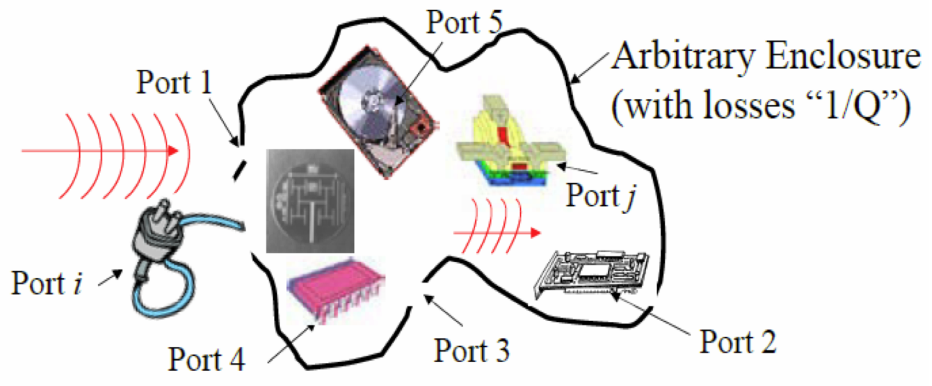
What can we say without solving the full electromagnetic problem?

Statistical theory can simplify modelling

Wave Chaos



Random Coupling Model (RCM)

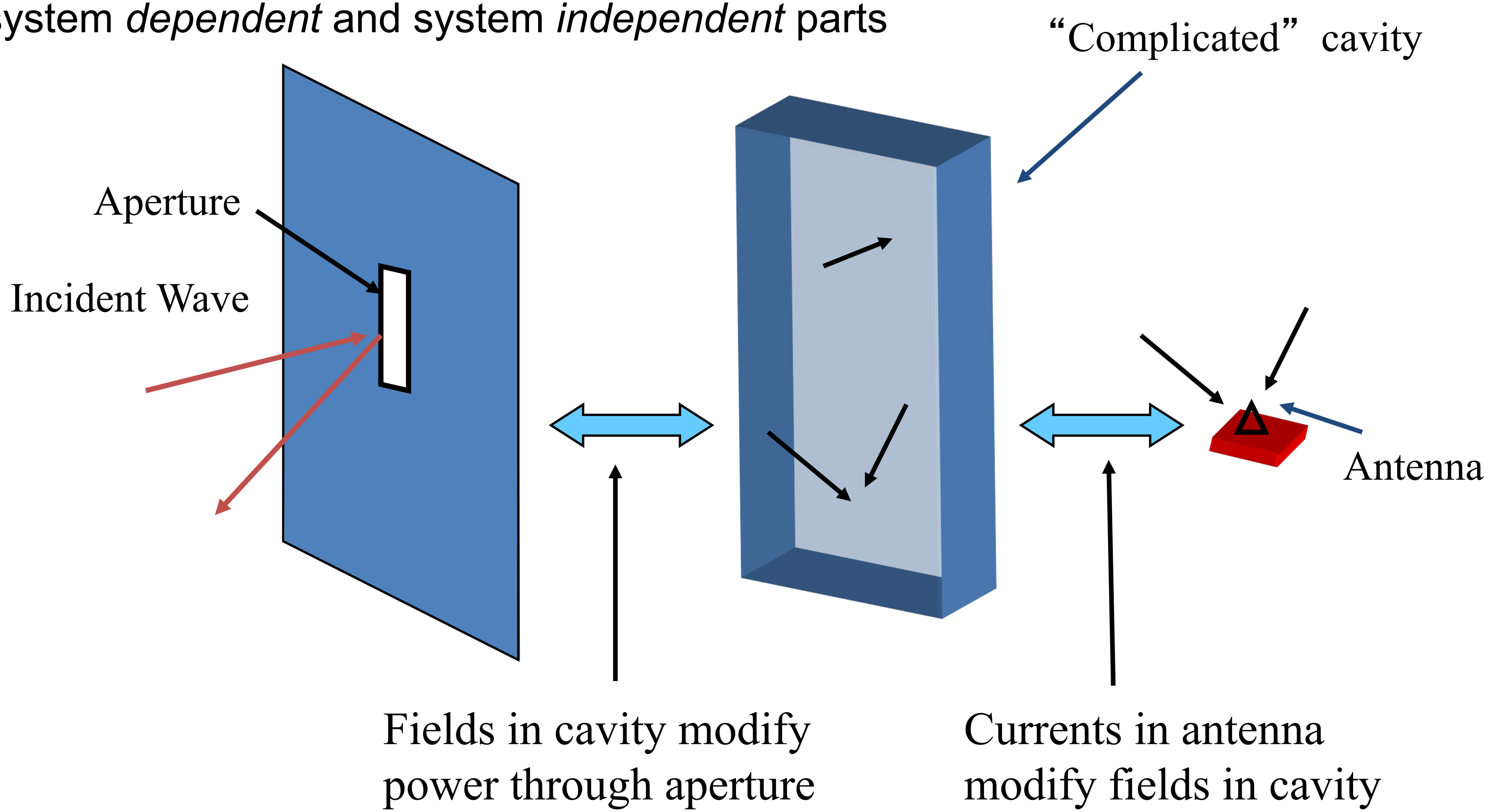


Hemmady, PRL 94, 014102 (2005)

- Most real life enclosures are ray-chaotic: small initial difference ➔ trajectories diverge exponentially in time.
- Wave properties are extremely sensitive to boundaries ➔ Wave Chaos
- Statistical tools are used to describe transmission in such chaotic enclosures
  - Random Matrix Theory – universal properties of **closed systems**
  - Random Coupling Model – incorporates system-specific properties

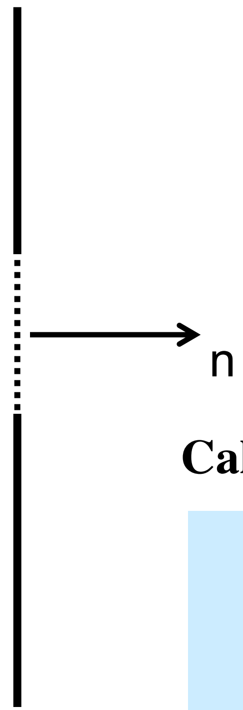
Random Coupling Model (RCM)

Break problems into component parts:
system *dependent* and system *independent* parts



Source: G. Gradoni et al, IEEE Trans EMC, vol. 57, issue 5, 2015.

RCM: System-specific part



1. Specify

$$\mathrm{E}_t = \sum_s V_s \mathrm{e}_s(\mathrm{x}_\perp)$$

2. Solve

$$-i\omega\varepsilon \mathrm{E}(\mathrm{x}) = \nabla \times \mathrm{H}(\mathrm{x})$$

$$i\omega\mu \mathrm{H}(\mathrm{x}) = \nabla \times \mathrm{E}(\mathrm{x})$$

3. Find

$$\mathrm{H}_t = -\sum_s I_s \mathrm{n} \times \mathrm{e}_s(\mathrm{x}_\perp)$$

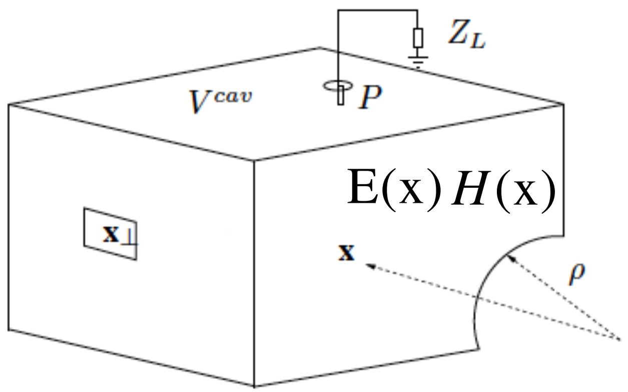
Calculate <u>admittance</u> matrix describing z>0.

$$I_s = \sum_{s'} Y_{ss'}^{rad}(k_0)\, V_{s'}$$

Sommerfeld radiation condition

$$Y_{ss'}^{rad}(k_0 = \omega / c) = \sqrt{\frac{\varepsilon}{\mu}} \int \frac{d^3k}{(2\pi)^3} \frac{2ik_0}{k_0^2 - k^2}\; \overline{\mathrm{e}}_s \cdot \underline{\underline{\Delta}} \cdot \overline{\mathrm{e}}_{s'}$$

Boundary value problem



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1. Formally expand fields of cavity in a basis of modes

$$\mathrm{E}(\mathrm{x}) = \sum_n V_n^{em} \mathrm{e}_n^{em}(\mathrm{x}) \qquad \mathrm{H}(\mathrm{x}) = \sum_n \left(I_n^{em} \mathrm{h}_n^{em}(\mathrm{x}) + I_n^{ms} \mathrm{h}_n^{ms}(\mathrm{x})\right)$$

2. Leads to <u>exact</u> expression for cavity admittance

$$Y_{ss'}^{cav}(k_0) = \sqrt{\frac{\varepsilon}{\mu}} \sum_n \left(\frac{ik_0}{k_0^2 - k_n^2} \frac{w_{sn}^{em} w_{s'n}^{em}}{V^{em}} + \frac{i}{k_0} \frac{w_{sn}^{ms} w_{s'n}^{ms}}{V^{ms}} \right) \qquad w_{sn}^{(.)} = \int_{aperture} d^2 x_\perp \;\; \mathrm{e}_s(\mathrm{x}_\perp) \cdot \mathrm{n} \times \mathrm{h}_n^{(.)}$$

High frequency regime

Large, linear eigenvalue problem

$$Y_{ss'}^{cav}(k_0) = \sqrt{\frac{\varepsilon}{\mu}} \sum_n \left(\frac{ik_0}{k_0^2 - k_n^2} \frac{w_{sn}^{em} w_{s'n}^{em}}{V^{em}} + \frac{i}{k_0} \frac{w_{sn}^{ms} w_{s'n}^{ms}}{V^{ms}} \right)$$

$$w_{sn}^{(.)} = \int_{aperture} d^2 x_\perp \;\; \mathrm{e}_s(\mathrm{x}_\perp) \cdot \mathrm{n} \times \mathrm{h}_n^{(.)}$$

Eigenvalues:
Random Matrix Theory

Eigenfunctions:
Random Wave Hypothesis

Alternative approach: statistical model for eigenvalues and eigenfunctions

Eigenfunction complexity: statistical approach

How to capture field complexity? Modal patterns alike at high frequencies!

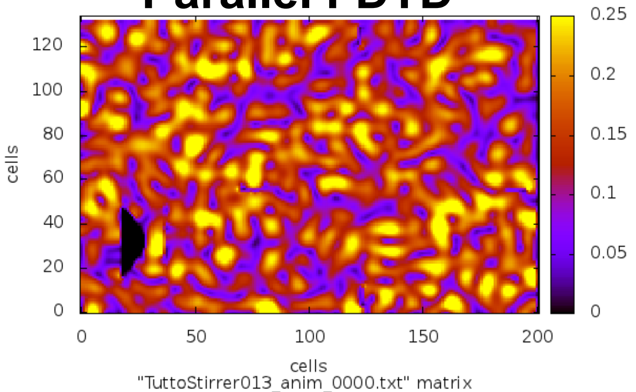
$$-\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)\psi_n = k_n^2\psi_n\,,$$

$$\psi_n|_S = 0$$

$$H\psi_n = k_n^2\psi_n$$

Hamiltonian (energy) operator

Parallel FDTD



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Eigenvalues:
Random Matrix Theory

Eigenfunctions:
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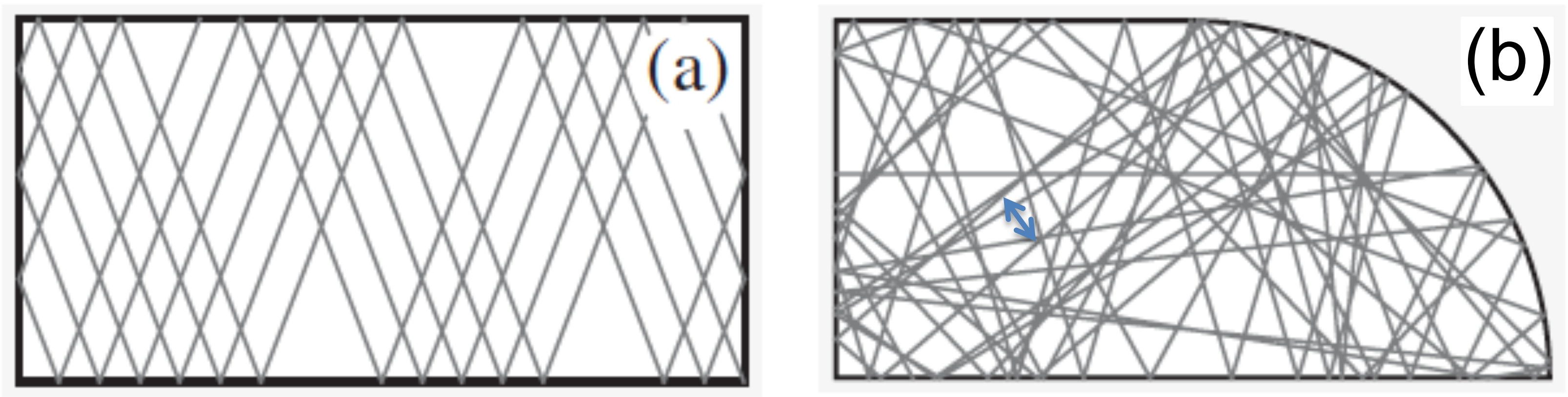
Classical Chaos

Simple 2D deformations lead to dynamical mixing



Lyapunov instability: divergent trajectories that are hard-to-predict, mixing.

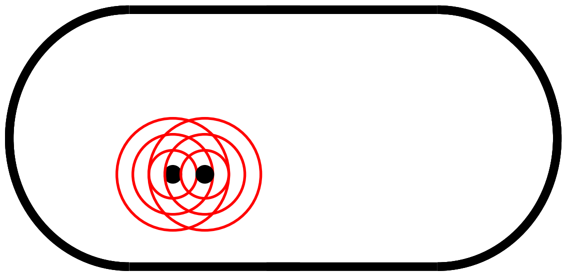
Theorem: (*Poincare'*) Perturbation lead to non-existence of integrals of motions, including energy.

$$\delta x_t = e^{\lambda t} \delta x_0$$

Lyapunov exponent

Wave Chaos

1) Waves do not have trajectories



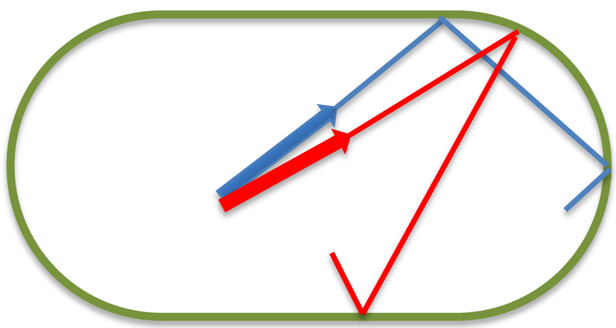
It makes no sense to talk about "diverging trajectories" for waves

2) Linear wave systems can't be chaotic

Maxwell's equations, Schrödinger's equation are linear

3) However in the semiclassical limit, you can think about rays:
it is possible to define chaos in the ray-limit

$$\delta x_0 \approx 0$$



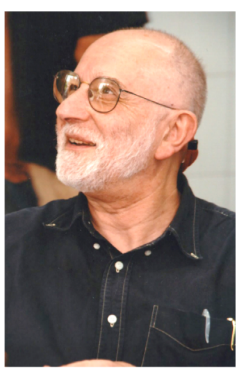
$$\delta x_t = e^{\lambda t} \delta x_0 >> 0$$

Wave Chaos concerns solutions of linear wave equations which, in the semiclassical limit, can be described by chaotic ray trajectories

From Ray chaos to Wave chaology

Quantum mechanics: Classical instability of trajectories results in complex *wavefunctions*

Sir M. V. Berry



Proc. R. Soc. Lond. A **413**, 183–198 (1987)
Printed in Great Britain

The Bakerian Lecture, 1987

Quantum chaology

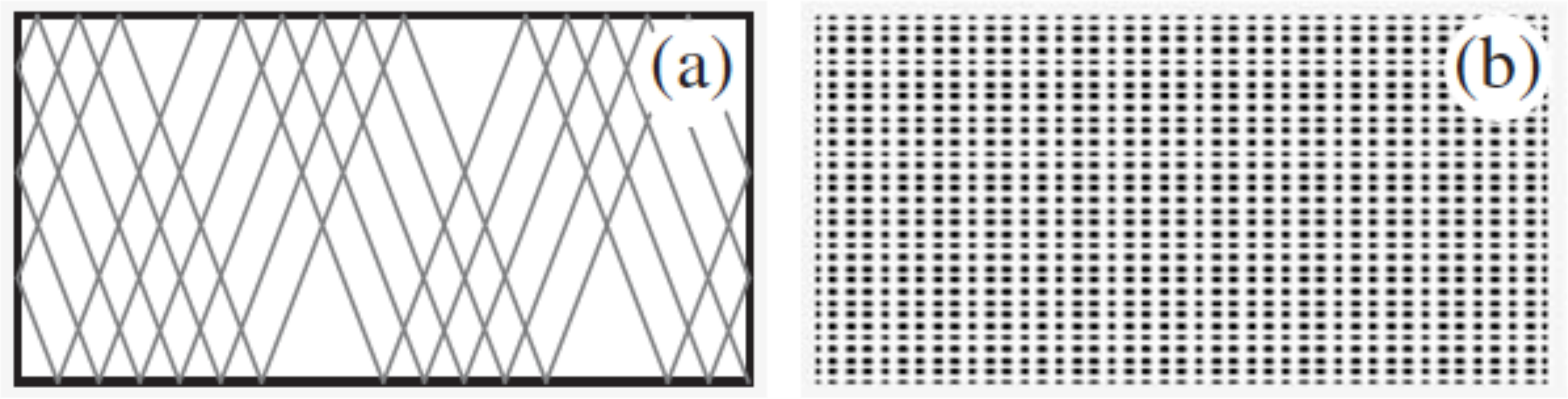
By M. V. Berry, F.R.S.
H. H. Wills Physics Laboratory, Tyndall Avenue, Bristol BS8 1TL, U.K.

(*Lecture delivered* 5 *February* 1987 – *Typescript received* 2 *March* 1987)

Bounded or driven classical systems often exhibit chaos (exponential instability that persists), but their quantum counterparts do not. Nevertheless, there are new régimes of quantum behaviour that emerge in the semiclassical limit and depend on whether the classical orbits are regular or chaotic, and this motivates the following definition.
Definition. Quantum chaology is the study of semiclassical, but non-classical, behaviour characteristic of systems whose classical motion exhibits chaos.

Integrable billiards

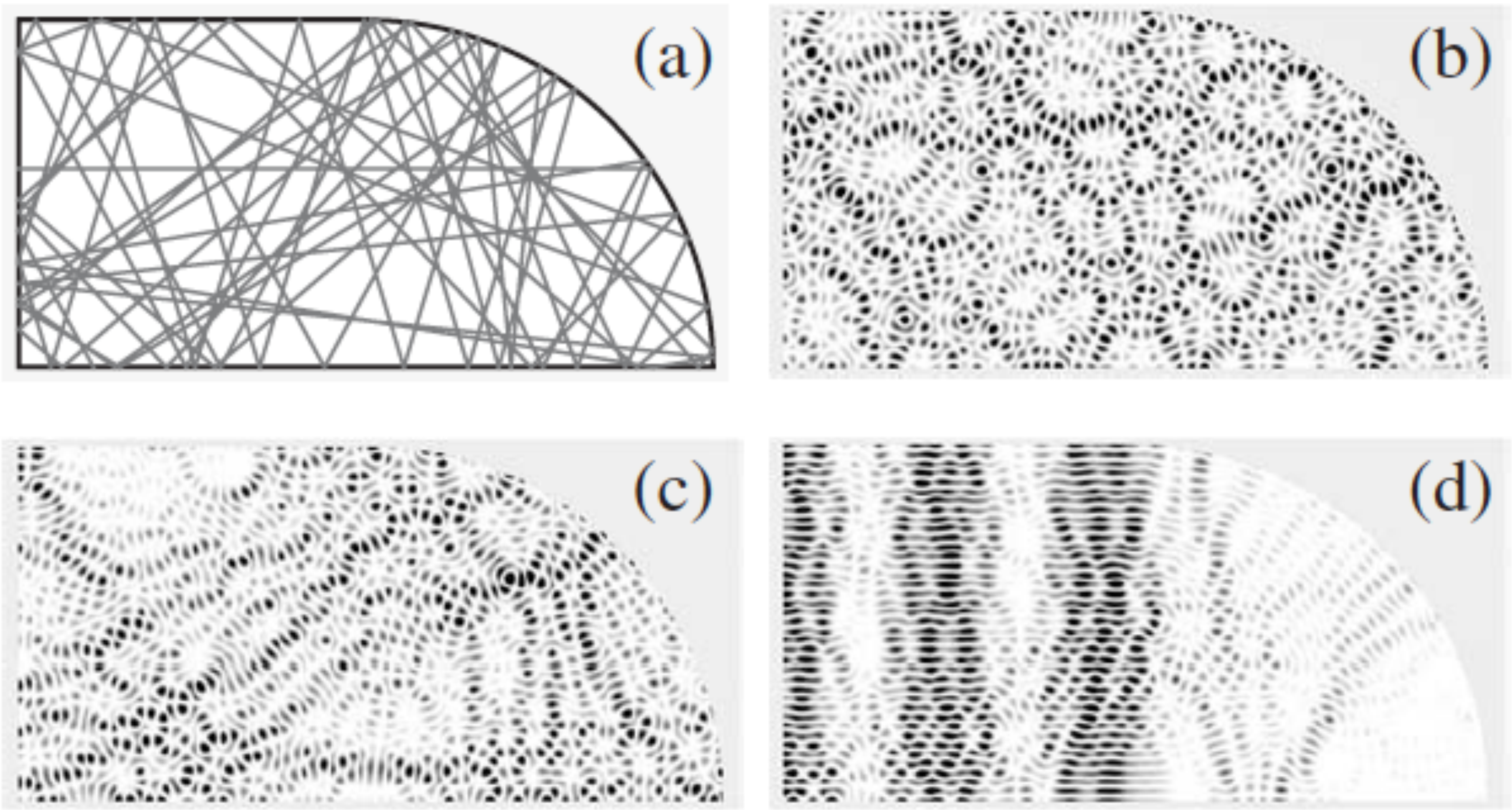
Is trajectory stability related to **eigenfunction** structure?



Question: what is the Lyapunov exponent of the rectangular billiard?

$$\lambda = 0$$

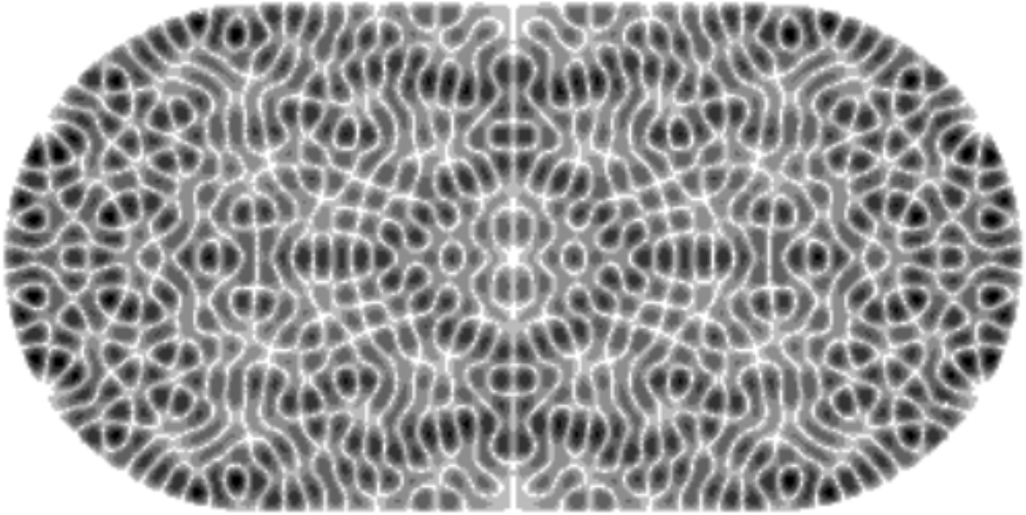
Non-Integrable billiards and wave chaos



Exact calculation hard, **eigenfunction** statistics accessible

Eigenvalue complexity: statistical approach

Stadium billiard



$$-\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)\psi_n = k_n^2\psi_n\,, \qquad \psi_n|_S = 0$$

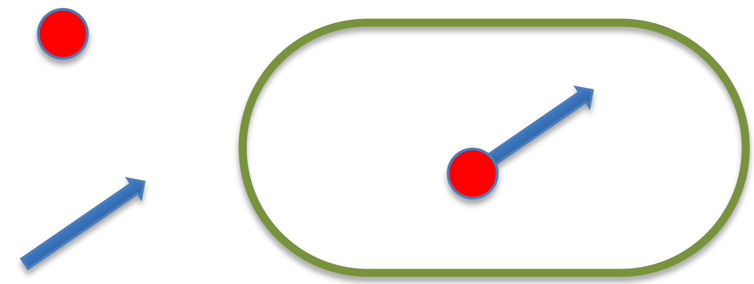
Eigenvalues:
Random Matrix Theory

Eigenfunctions:
Random Wave Hypothesis

Phase-space and Wigner function

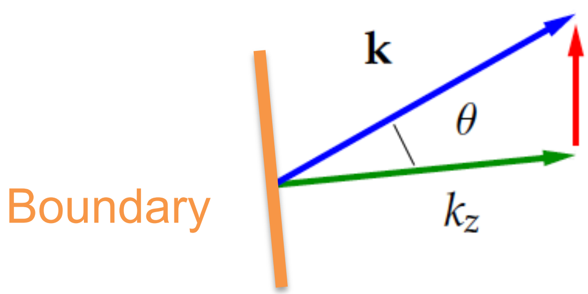
Classical phase-space represents particle state space

$$\Omega = ? \quad \begin{cases} \text{Position, } \boldsymbol{r} \\ \text{Momentum, } \boldsymbol{p} \end{cases}$$

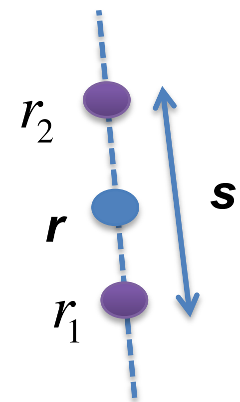


Wigner function represents waves in a dynamical (state) space

$$W(\Omega) \equiv W(r,p) = \left(\frac{k}{2\pi}\right)^d \int e^{ikps} C\left(r + \frac{s}{2}, r - \frac{s}{2}\right) ds$$



$k \sin\theta = kp$



Combined positional and directional information on wave/ray density propagation

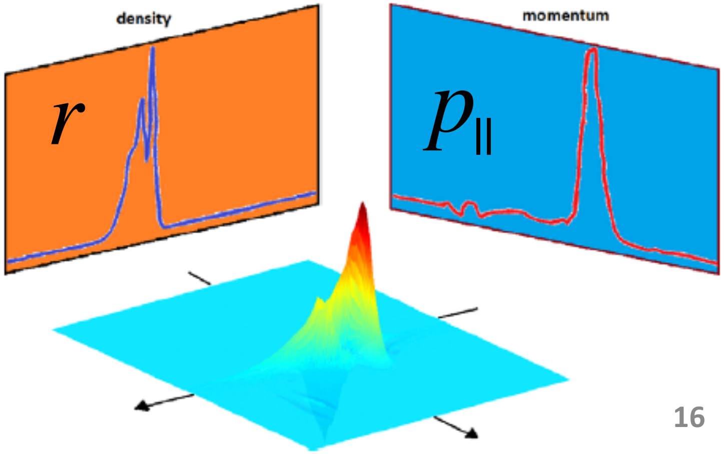
Wave phase-space

Berry's idea: study wave phase-space structure to unfold eigenfunction statistics

$$C_{nm}(r_1, r_2) \approx \left\langle \psi_n(r_1)\psi_n^*(r_2) \right\rangle$$

Quantum/wave phase-space is constructed through **Wigner function**

$$W(r,p) = \int_D e^{ikps} C_{nm}\left(r = \frac{r_1 + r_2}{2}, s = r_1 - r_2\right) ds$$



Position ***r*** = location of trajectories
Momentum ***p*** = directions of propagation

Eigenfunction Statistics

Hypothesis: (Voros, Berry) The Wigner function of *ergodic* systems is constant in the energy shell as it is formed by a continuum of momenta *p*

$$W(r,p) = \frac{1}{V_\Omega}\delta\left(E - H(r,p)\right)$$

Correlation function obtained by inverse Fourier transform

$$C_{nm}(r,s) = \frac{1}{|\psi(0)|^2}\int_\Pi e^{-ikps} W_{nm}(r,p)dp = \frac{1}{V_\Omega}\int_\Pi e^{-iksp}\delta\left(E - H(r,p)\right)dp$$

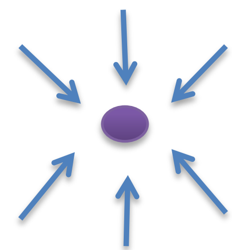
$$= J_0(ks), \quad 2D$$

$$= \frac{\sin(ks)}{ks}, \quad 3D$$

Random Wave Hypothesis

Theorem: (Berry) Ergodic wave-functions are Gaussian random variables reproduced by random plane wave superposition

$$\psi_m(r) = \frac{1}{\sqrt{N}} \sum_{n=1}^{N} a_n \exp\left(ik_n \cdot r + \chi_n\right)$$



$$N \to \infty$$

CLT: Zero mean **GRV**

Source: *Berry*, Regular and irregular wave-functions, JPA, 1977

The autocorrelation function is not by itself sufficient to determine all statistical properties of ψ. However it is likely that for stochastic classical motion the phases of the different contributions $\boldsymbol{p}$ to ψ are uncorrelated, because the orbit would accumulate many action units $\hbar$ in its 'unpredictable' wanderings between passages through the neighbourhood of $\boldsymbol{q}$. This would imply that ψ is a *Gaussian random function* of $\boldsymbol{q}$ (Rice 1944, 1945, Longuet-Higgins 1956), whose spectrum at $\boldsymbol{q}$ is simply the local average of the Wigner function $\bar{\Psi}(\boldsymbol{q}, \boldsymbol{p})$. For ergodic motion $\bar{\Psi}$ is

Ergodic eigenfunctions

Berry's hypothesis: m-th mode represented by N plane waves

$$\psi_m(r) = \frac{1}{\sqrt{N}} \sum_{n=1}^{N} a_n \exp\left(ik_n \cdot r + \chi_n\right)$$

Find eigenfunction statistics from random plane wave superposition

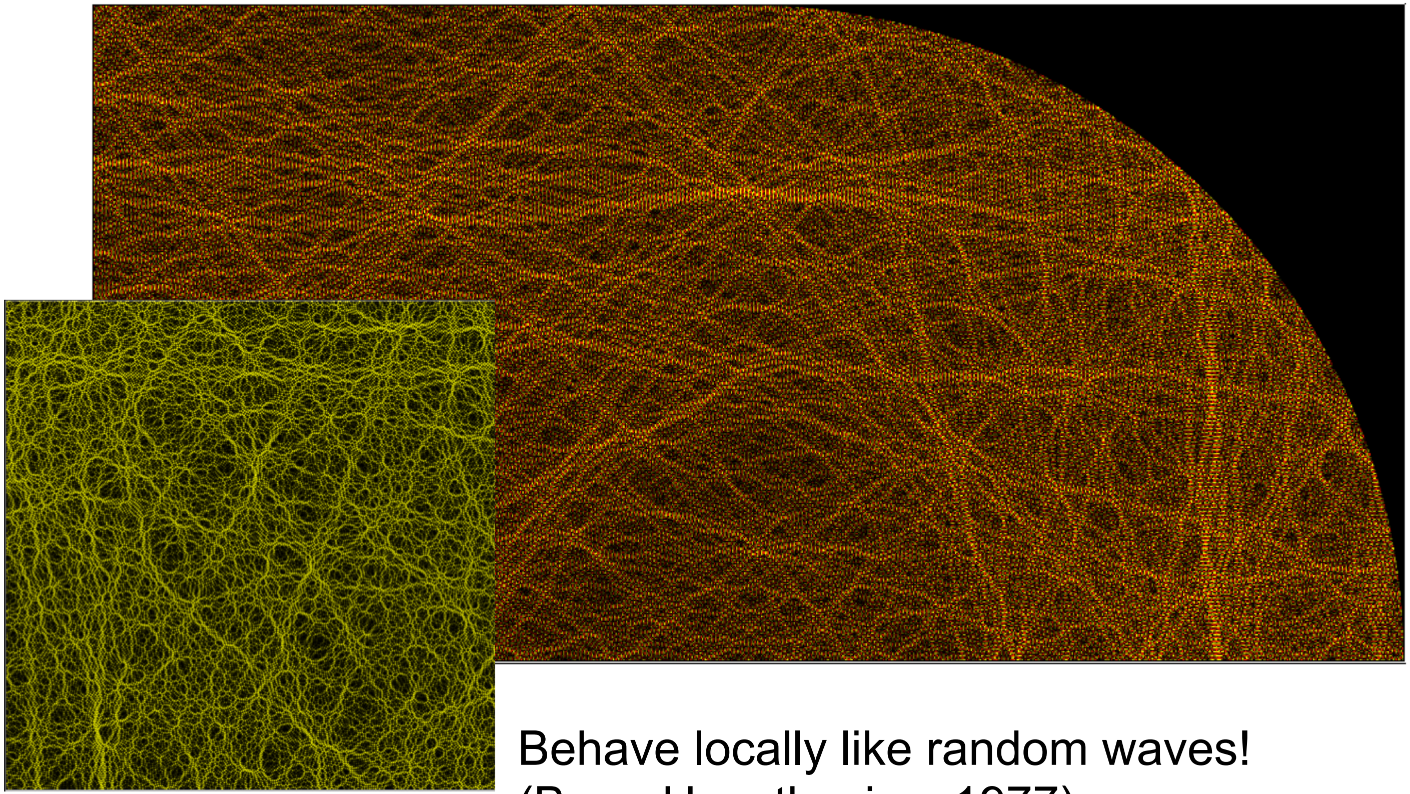
$$\begin{aligned} C_{ms}(r_1, r_2) &= \left\langle \psi_m(r_1)\psi_s^*(r_2) \right\rangle \\ &= J_0\left(k(r_1 - r_2)\right), \quad 2D \\ &= \frac{\sin\left(k(r_1 - r_2)\right)}{k(r_1 - r_2)}, \quad 3D \end{aligned}$$

Eigenfunctions in Stadium Billiards

Eigenfunctions are (on average) equally distributed –

(K) Mixing ⇒ Ergodicity

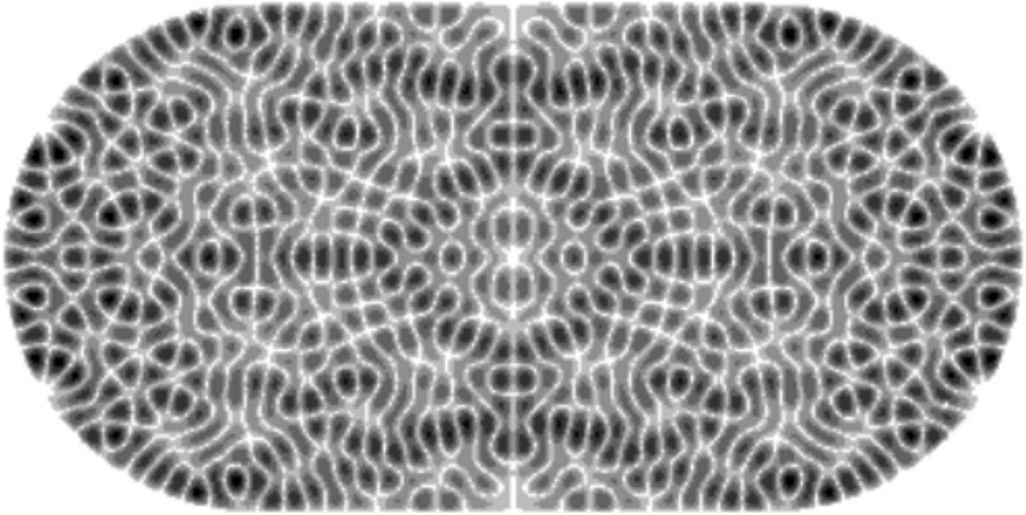
Source: Alex Barnett



Behave locally like random waves!
(Berry Hypothesis – 1977)

Eigenvalue complexity: statistical approach

Stadium billiard



$$-\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)\psi_n = k_n^2\psi_n\,, \qquad\qquad \psi_n|_S = 0$$

Eigenvalues:
Random Matrix Theory

Eigenfunctions:
Random Wave Hypothesis

Statistics of spectra

Bohigas, *Giannoni*, *Shmitt* conjecture (1984)

BGS conjecture : *"Spectra of time-reversal-invariant systems whose classical analogs are K systems show the same fluctuation properties as predicted by GOE (alternative stronger conjectures that cannot be excluded would apply to less chaotic systems, provided that they are ergodic). If the conjecture happens to be true, it will then have been established the* universality of the laws of level fluctuations *in quantal spectra already found in nuclei and to a lesser extent in atoms. Then, they should also be found in other quantal systems, such as molecules, hadrons, etc."*. The paper had been extre-

Dynamical system properties:

(K) Mixing $\Longrightarrow$ Ergodicity

Scroedinger vs Helmoltz

$$-\frac{\hbar^2}{2m}\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)\psi_n = E_n\psi_n\,. \qquad -\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)\psi_n = k_n^2\psi_n\,,$$

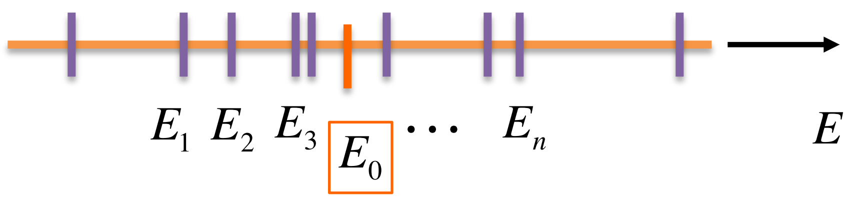
$$\psi_n|_S = 0$$

Source: Stoeckmann. Seminaire Poincare' 2006

Statistics of Spectra

Eigenvalue interaction has an imprint of the classical dynamics

Sequence of Nearest Neighbor Spacings (NNS) $E_n \equiv k_n^2$



$$\{E_n\} \longrightarrow \Delta E_n = \{E_{n+1} - E_n\} \quad n = 1 \ldots N-1$$

$$s_n = \frac{\Delta E_n}{\langle \Delta E \rangle}$$

Gap distribution?

(Wigner's) Random Matrix Theory

NNS shows universal statistical fluctuations!

Theorem: (*Wigner***)** Probability distribution of NNS s_n belongs to universality classes established by symmetries (e.g., time-reversal, parity).

GOE *Wigner* Surmise:

$$\Pr(s) = \frac{\pi}{2} s \exp\left(-\frac{\pi}{4} s^2\right)$$

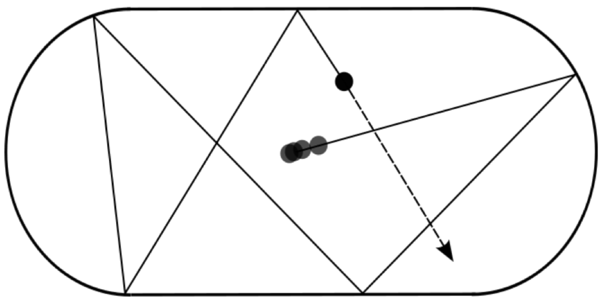
Theorem: (*Berry-Tabor***)** Generic regular systems have randomly distributed eigenenergies with Poisson distributed NNS

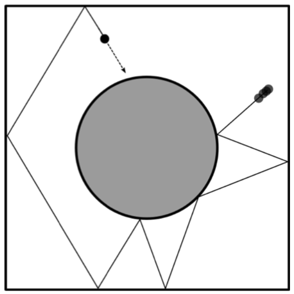
Regular *Berry* Surmise:

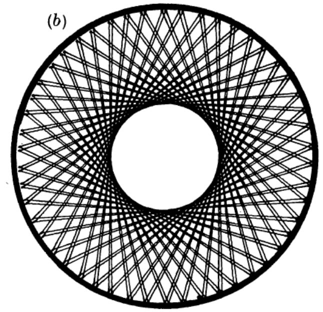
$$\Pr(s) = \exp(-s)$$

Random Matrix Theory

Stadium, Sinai-Lorentz, Circle: we know their classical dynamics!



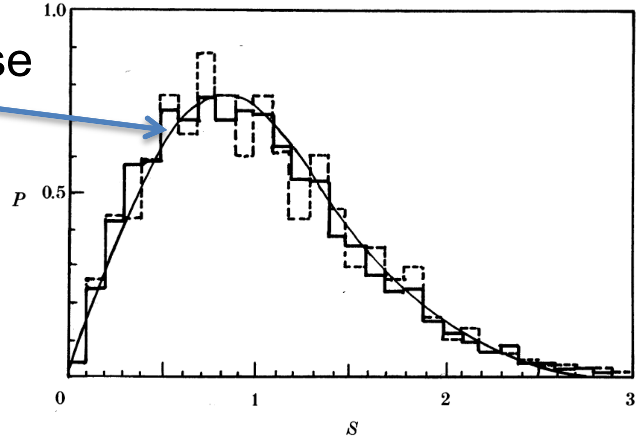


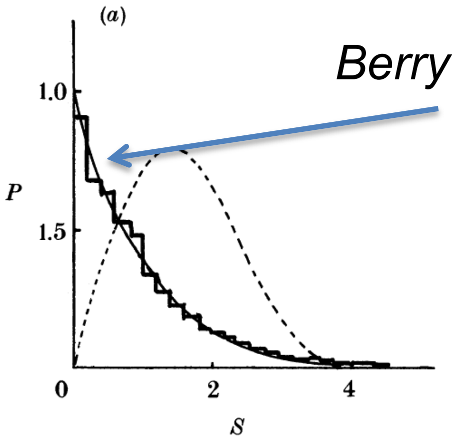


?

What are their **eigenvalue gap (spacing)** distributions?

Wigner surmise





Berry surmise

``This is universality!''

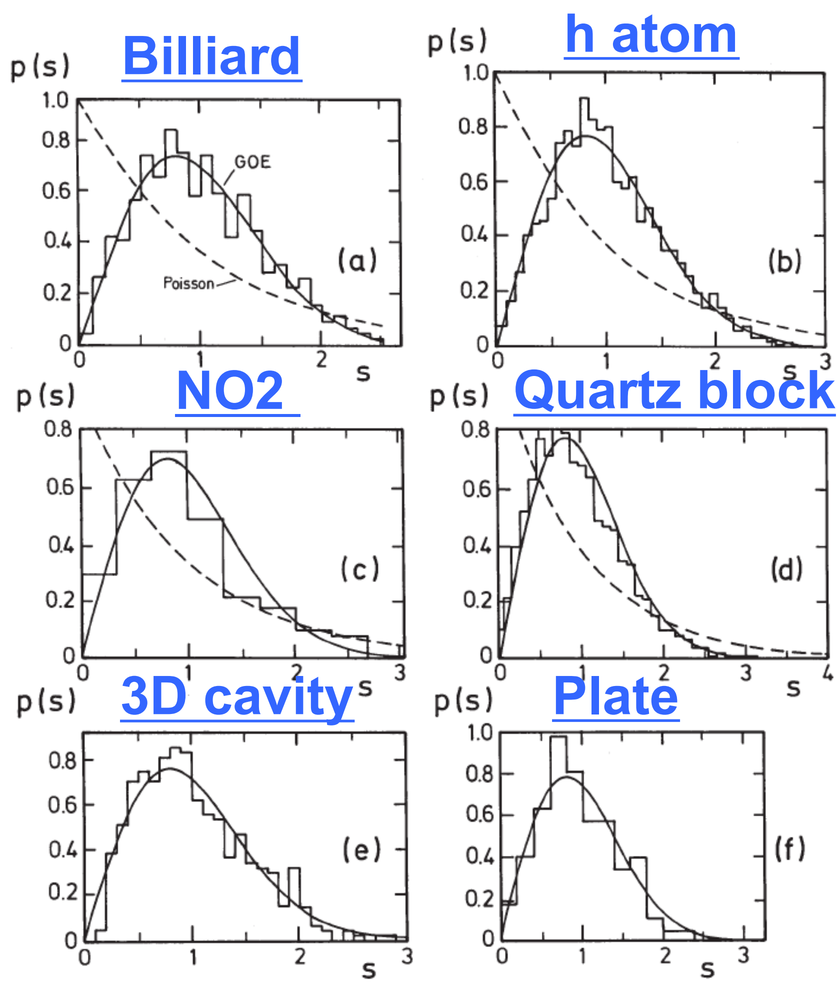
To see the universality a magnification factor is needed:

$$\langle \rho \rangle = k^d \frac{dV_{\Omega(E)}}{dE}$$

$$\downarrow$$

$$\langle \Delta E \rangle \sim \rho^{-1} \sim k^{-d}$$

$$s_n = \frac{\Delta E_n}{\langle \Delta E \rangle}$$



Credit: *Stoeckmann*

Boundary value problem

1. **Formally expand fields of cavity in a basis of modes**

$$\mathrm{E}(\mathrm{x}) = \sum_n V_n^{em} \mathrm{e}_n^{em}(\mathrm{x}) \qquad \mathrm{H}(\mathrm{x}) = \sum_n (I_n^{em} \mathrm{h}_n^{em}(\mathrm{x}) + I_n^{ms} \mathrm{h}_n^{ms}(\mathrm{x}))$$

2. **Leads to <u>exact</u> expression for cavity admittance**

$$Y_{ss'}^{cav}(k_0) = \sqrt{\frac{\varepsilon}{\mu}} \sum_n \left(\frac{ik_0}{k_0^2 - k_n^2} \frac{w_{sn}^{em} w_{s'n}^{em}}{V^{em}} + \frac{i}{k_0} \frac{w_{sn}^{ms} w_{s'n}^{ms}}{V^{ms}} \right) \qquad w_{sn}^{(.)} = \int_{aperture} d^2 x_\perp \ \mathrm{e}_s(\mathrm{x}_\perp) \cdot \mathrm{n} \times \mathrm{h}_n^{(.)}$$

3. **Replace exact modes with "chaotic" modes (random plane wave superposition)**

$$\mathrm{h}_{n\perp}^{em} = \lim_{N\to\infty} \frac{2}{\sqrt{N}} \sum_{j=1}^{N} \mathrm{b}_{j\perp} \cos(\mathrm{k}_j \cdot \mathrm{n}\, z) \cos(\theta_j + \mathrm{k}_j \cdot \mathrm{x}_\perp)$$

4. **Replace exact spectrum with RMT spectrum, add damping**

$$k_0^2 \to k_0^2 \left(1 + i \frac{1}{Q} \right)$$

Coupling coherence matrix

An interesting connection between free-space and Boundary Value Problem is found

$$\frac{w^{em}_{sn} w^{em}_{s'n}}{V^{em}} = 2\frac{\Delta k^2}{\eta_0}\left\{\left[\mathrm{Re}\left(\underline{\underline{Y}}^{rad}\right)\right]^{1/2}\cdot \underline{\Phi}_n \underline{\Phi}_n^T \cdot \left[\mathrm{Re}\left(\underline{\underline{Y}}^{rad}\right)\right]^{1/2}\right\}_{ss'}$$

C_{ms}

$$\frac{i}{\eta_0 k_0}\sum_n \frac{w^{ms}_{sn} w^{ms}_{s'n}}{V^{ms}} \approx \frac{i}{\eta_0 k_0}\sum_n \left\langle \frac{w^{ms}_{sn} w^{ms}_{s'n}}{V^{ms}} \right\rangle \approx iB^{ms}_{ss'}$$

Plugging back and rearranging terms yield the decomposition form

$$\underline{\underline{Y}}^{cav} = i\,\mathrm{Im}\left(\underline{\underline{Y}}^{rad}\right) + \left[\mathrm{Re}\left(\underline{\underline{Y}}^{rad}\right)\right]^{1/2}\cdot \underline{\underline{\xi}} \cdot \left[\mathrm{Re}\left(\underline{\underline{Y}}^{rad}\right)\right]^{1/2}$$

Random matrix (normalised impedance) whose elements are Renormalized Green's functions of the closed cavity

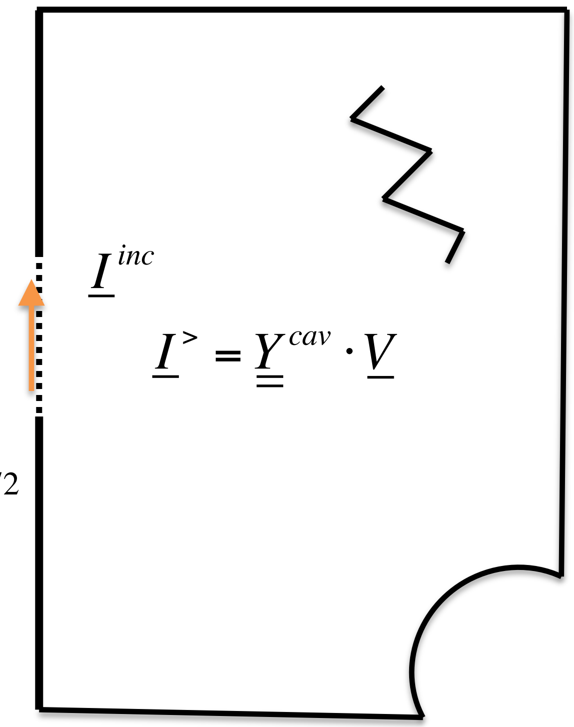
Source: Gradoni *et al*, IEEE Trans EMC, vol. 57, issue 5, 2015.

RCM for radiation problems

$$\left(\underline{\underline{Y}}^{cav} + \underline{\underline{Y}}^{rad}\right)\cdot \underline{V} = 2\underline{I}^{inc}$$

$$P_t = \frac{1}{2}\mathrm{Re}\left(\underline{V}^* \cdot \underline{\underline{Y}}^{cav} \cdot \underline{V}\right)$$

$$\underline{I}^{<} = -\underline{\underline{Y}}^{rad}\cdot \underline{V}$$



Cavity impedance from RCM

$$\underline{\underline{Y}}^{cav} = i\,\mathrm{Im}\left(\underline{\underline{Y}}^{rad}\right) + \left[\mathrm{Re}\left(\underline{\underline{Y}}^{rad}\right)\right]^{1/2}\cdot \underline{\underline{\xi}} \cdot \left[\mathrm{Re}\left(\underline{\underline{Y}}^{rad}\right)\right]^{1/2}$$

$$\underline{\underline{Y}}^{rad} = i\,\mathrm{Im}\left(\underline{\underline{Y}}^{rad}\right) + \mathrm{Re}\left(\underline{\underline{Y}}^{rad}\right)$$

Coupling coefficients are GRVs

$$\left(\underline{\underline{\xi}}\right)_{ps} \approx -\frac{i}{\pi}\sum_{m=1}^{M}\frac{\varphi_{pm}\cdot\varphi_{sm}}{\dfrac{k_0^2 - k_m^2}{\left\langle \Delta k_m^2 \right\rangle} + i\alpha}$$

RMT spectrum

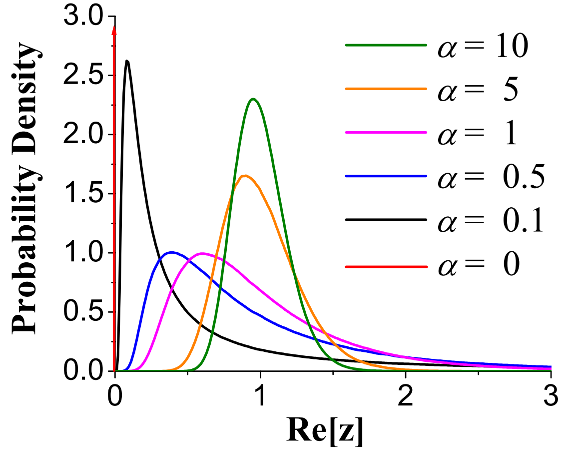
Modal overlapping!

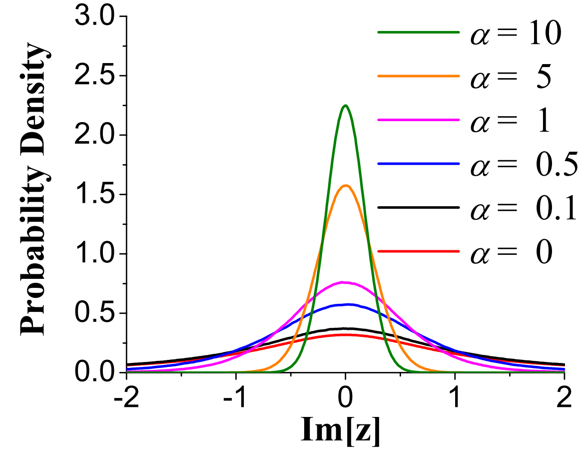
$$\alpha = \frac{k^2}{Q\Delta k^2} = \frac{B_Q}{\Delta k^2}$$

Distribution of cavity impedance

An example of Monte Carlo computation…

$$z \approx \left(\underline{\underline{\xi}}\right)_{11}$$ Diagonal normalized impedance



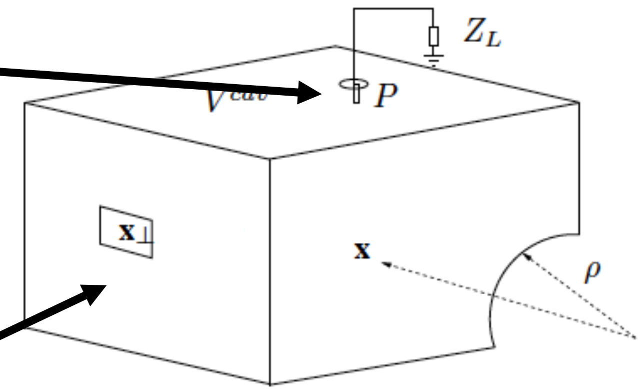


High loss: Gaussian distribution. Zero-loss: Lorenzian (Cauchy) distribution

GG *et al.* Wave Motion, 51-4, pp. 606-21, 2014

RCM for coexisting apertures and antennas

$$\underline{\underline{Z}}^{cav} = i\,\mathrm{Im}\left(\underline{\underline{Z}}^{rad}\right) + \left[\mathrm{Re}\left(\underline{\underline{Z}}^{rad}\right)\right]^{1/2} \cdot \underline{\underline{\xi}} \cdot \left[\mathrm{Re}\left(\underline{\underline{Z}}^{rad}\right)\right]^{1/2}$$



$$\underline{\underline{Y}}^{cav} = i\,\mathrm{Im}\left(\underline{\underline{Y}}^{rad}\right) + \left[\mathrm{Re}\left(\underline{\underline{Y}}^{rad}\right)\right]^{1/2} \cdot \underline{\underline{\xi}} \cdot \left[\mathrm{Re}\left(\underline{\underline{Y}}^{rad}\right)\right]^{1/2}$$

Radiation problem: Find unknown aperture currents and antenna voltages

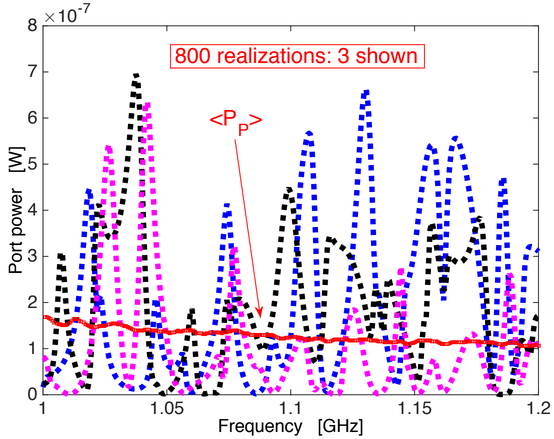
$$\underline{\underline{T}} = j\,\mathrm{Im}[\underline{\underline{U}}] + [\underline{\underline{V}}]^{1/2} \cdot \underline{\underline{\xi}} \cdot [\underline{\underline{V}}]^{1/2}$$

$$\begin{pmatrix} I_{aperture} \\ V_{antenna} \end{pmatrix} = \underline{\underline{T}} \begin{pmatrix} V_{aperture} \\ I_{antenna} \end{pmatrix}$$

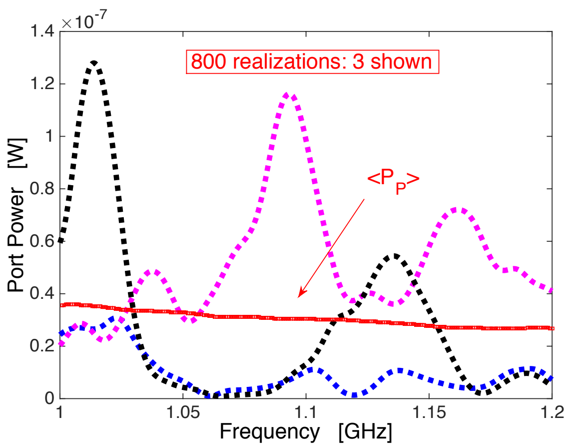
$$\underline{\underline{U}} = \begin{bmatrix} \underline{\underline{Y}}^{rad} & \underline{\underline{0}} \\ \underline{\underline{0}} & \underline{\underline{Z}}^{rad} \end{bmatrix} \qquad [\underline{\underline{V}}]^{1/2} = \begin{bmatrix} [\underline{\underline{Y}}^{rad}]^{1/2} & \underline{\underline{0}} \\ \underline{\underline{0}} & [\underline{\underline{Z}}^{rad}]^{1/2} \end{bmatrix}$$

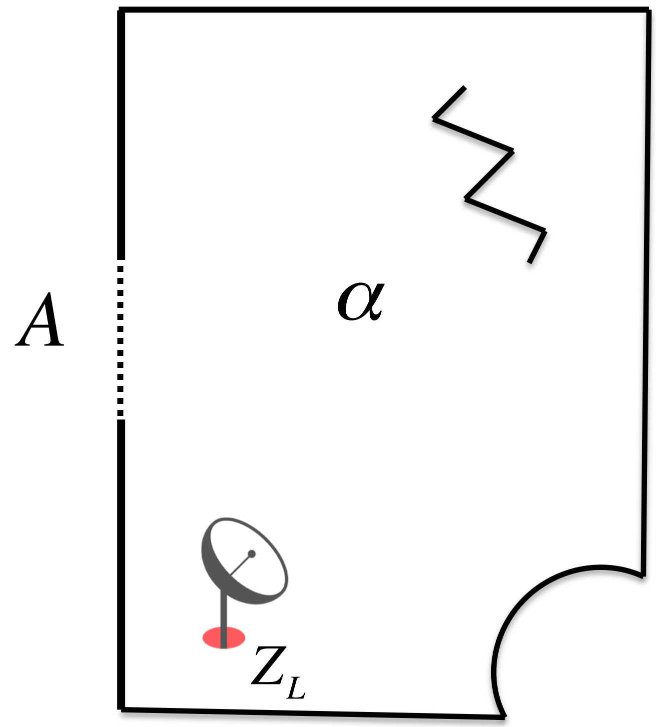
Power delivered to a terminal

$$\alpha = 0.1$$



$$\alpha = 1.0$$



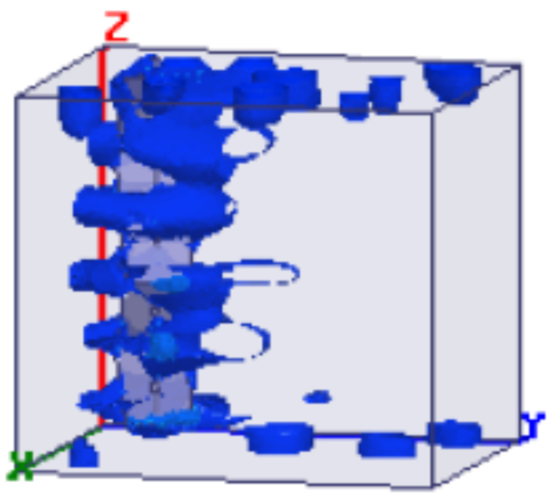


$$P_A = \frac{1}{2}\Re\left\{ \underline{I}^{inc,*} \cdot \left(\underline{\underline{Y}}^{rad} \right)^{-1,*} \cdot \underline{I}^{inc} \right\}$$

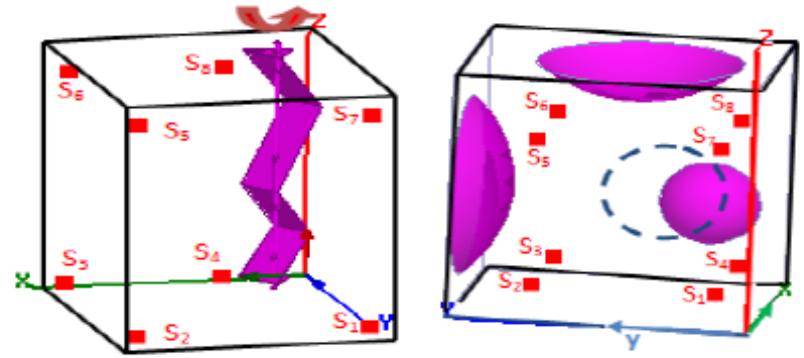
$$\langle P_L \rangle \approx \frac{R_L R^{rad}}{\left| Z_L + Z^{rad} \right|^2} \frac{P_A}{2\pi\alpha}$$

Why deforming an RC?

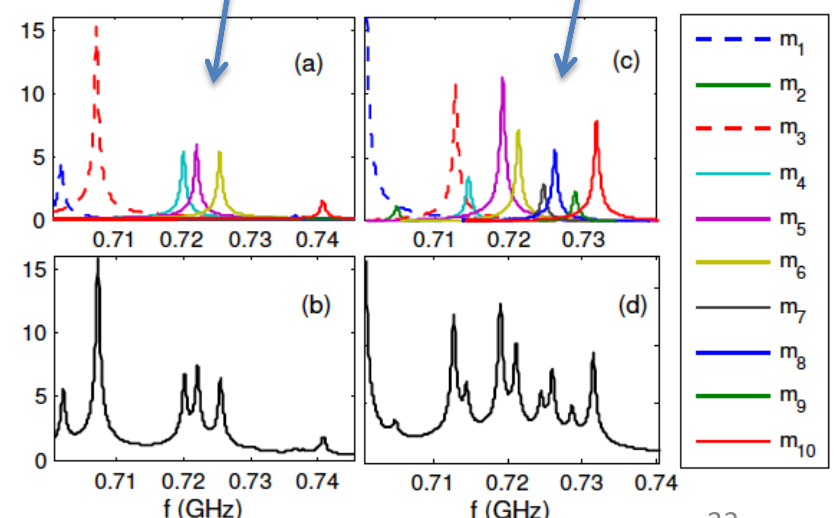
Curved diffractors diffuse energy
and suppress localised eigen-modes



205th mode (908 MHz),
Iso-amplitude surfaces



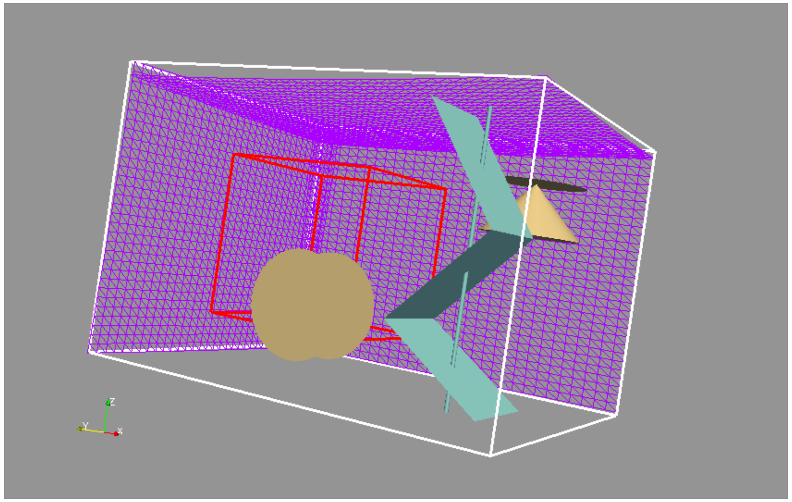
(a) Classical RC (C1)   (b) Chaotic cavity (C2)

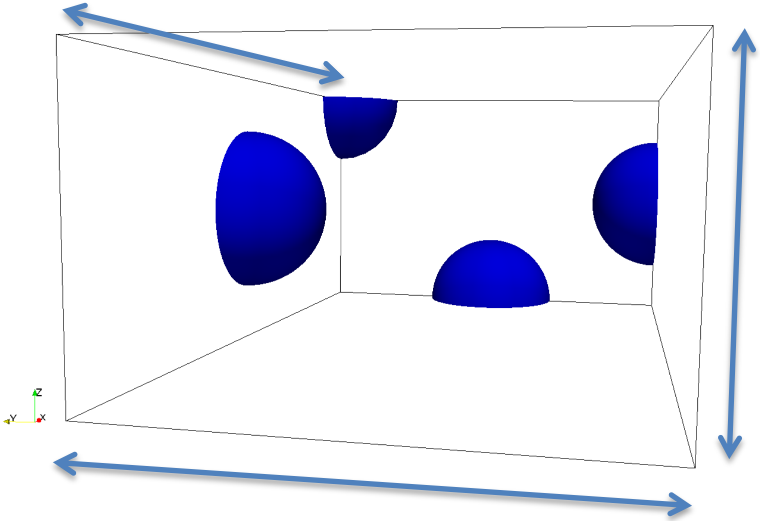


Source: Selemani et al 59, p.325 TEMC 2017

UoN-UnivPM Chaotic RC

Suppress localization by tilting walls: polygonal billiards



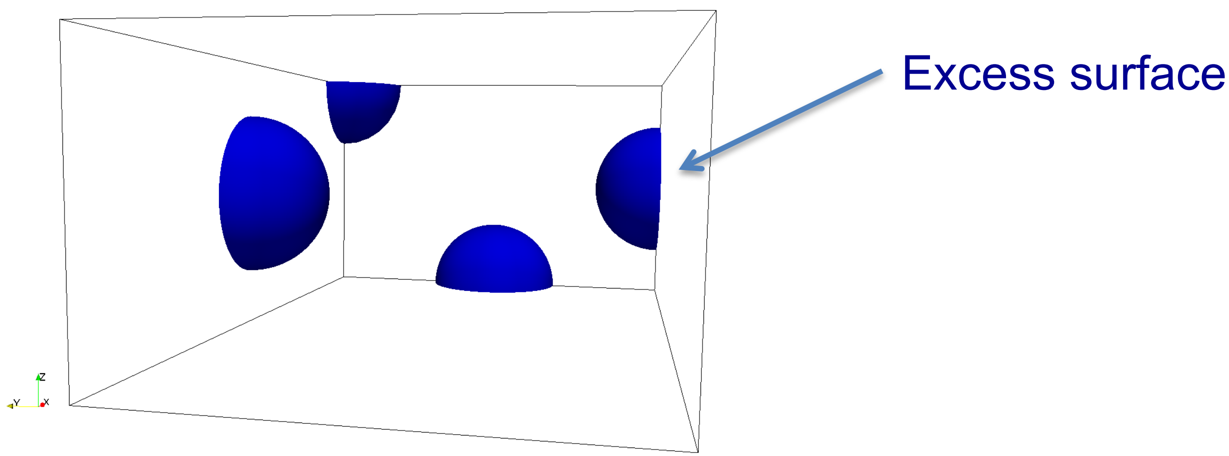


Need of design laws for large the chaotic RC

Source: **Luca Bastianelli**, Franco Moglie, Valter Mariani Primiani

Wave Control: Modal density

Local Modal density can be **increased** by boundary deformation



Local eigenmode denisty, good antenna coupling and matching:

$$M(f) = \langle \rho \rangle = \frac{1}{2\pi T(f)} \qquad T(f) \approx \left\langle \left| S_{21} \right|^2 \right\rangle \qquad \frac{M_c}{M_r} \cong \frac{\left\langle \left| S_{21,R} \right|^2 \right\rangle}{\left\langle \left| S_{21,C} \right|^2 \right\rangle}$$

Average modal density

RCM-based calculation of scattering statistics

$$S_{21} = \frac{2Z_{21}\left(R_{01}R_{02}\right)^{1/2}}{\left(Z_{11}+Z_{01}\right)\left(Z_{22}+Z_{02}\right)-Z_{12}Z_{21}}$$

In presence of modal overlapping (reverberation), $\alpha > 1$ using first-order Perturbation theory

$$S_{21} \approx \left[\frac{2\left(R_{01}R_{02}\right)^{1/2}\left(Z_{22}^{rad}\right)^{1/2}\left(Z_{22}^{rad}\right)^{1/2}}{\left(Z_{11}^{rad}+Z_{01}\right)\left(Z_{22}^{rad}+Z_{02}\right)}\right]\xi_{\alpha}$$

Assuming identical antennas

$$\frac{M_c}{M_r} \xrightarrow[\alpha>1]{} \frac{\left\langle\left|\xi_{\alpha,R}\right|^2\right\rangle}{\left\langle\left|\xi_{\alpha,C}\right|^2\right\rangle} = \frac{\alpha_C}{\alpha_R}$$

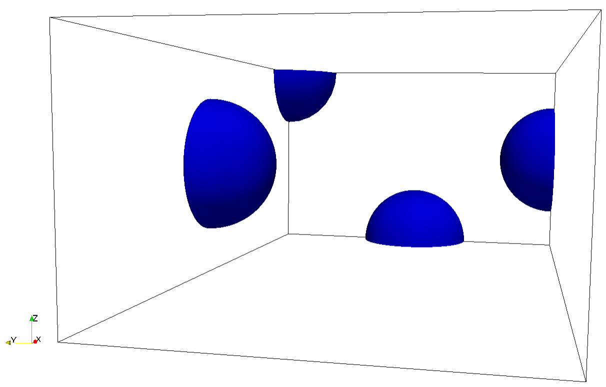
Excess surface geometry

Unfolding, we get a simple scaling law

$$\frac{M_c}{M_r} \cong 1 + \frac{S_{excess}}{S}$$

where the excess surface can be calculated from the geometry of deformations

$$S_{excess} = \pi \sum_{n=1}^{N} \beta_i r_i \qquad \beta_i = 4\beta_{si} - \beta_{ci}$$



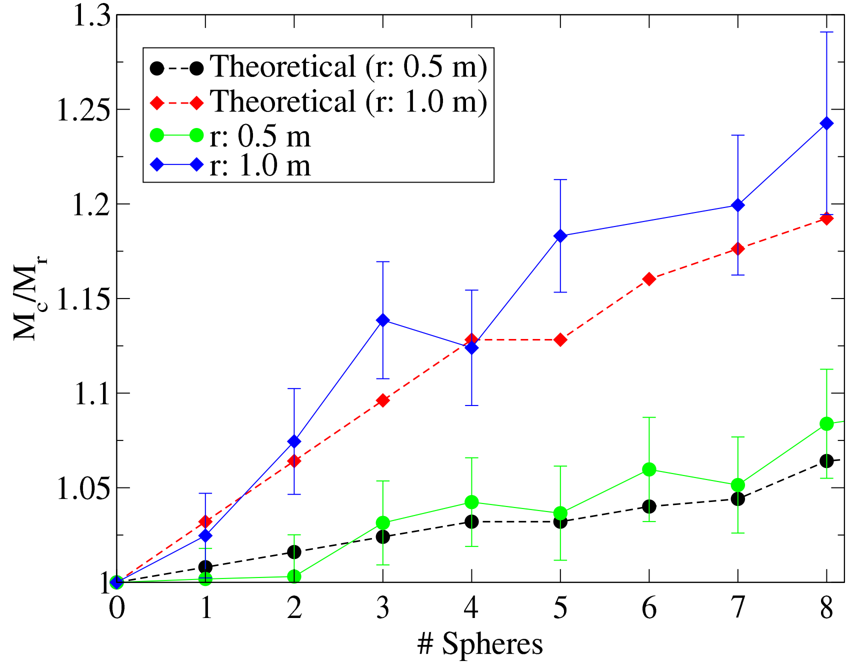
$$\beta_{si} = \begin{cases} \frac{1}{8} & \text{corner} \\ \frac{1}{4} & \text{edge} \\ \frac{1}{2} & \text{wall} \end{cases} \qquad \beta_{ci} = \begin{cases} \frac{3}{4} \\ 1 \\ 1 \end{cases}$$

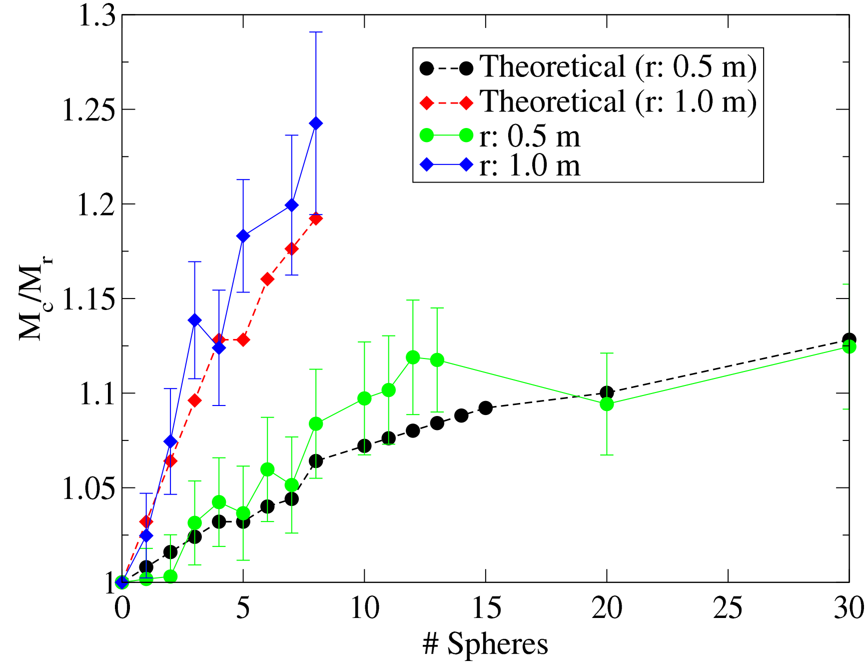
Physical bounds

Mode density ratio for *N* identical spheres of radius *r*

$$\frac{M_c}{M_r} \cong 1 + \frac{S_{excess}}{S}$$

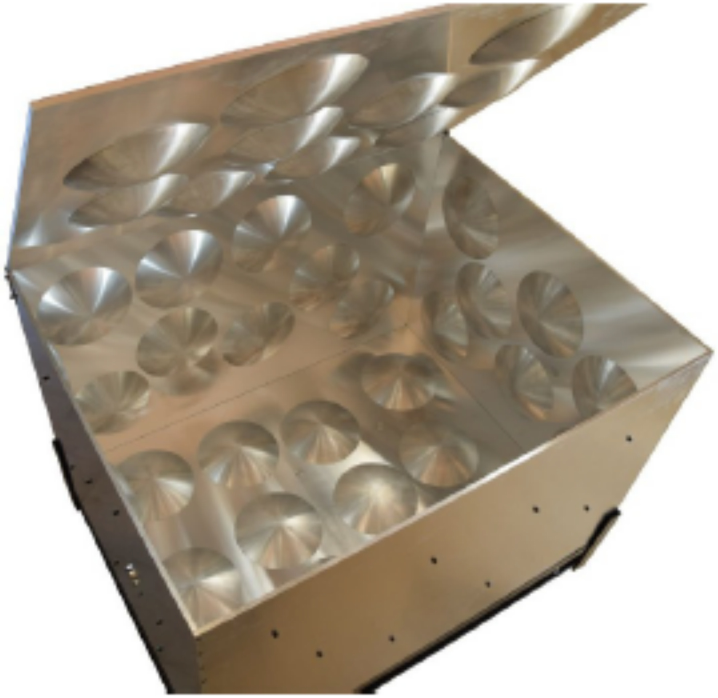
Theory vs **FDTD**





Chaotic RC @ UniNice, France

Spherical caps



Summary

Classical dynamics and stability associated to EM wave system:

- Simple deformations create *non-integrable* systems
- High frequency *ray instability* support wave chaos
- Random plane wave statistics constructed from semi-classical phase-space

Spectral footprint of Chaos:

- Classical orbit instability change resonance *gap* distribution
- *Random Matrix Theory* capture resonance statistics
- *Random Coupling Model* includes coupling and antenna EM radiation

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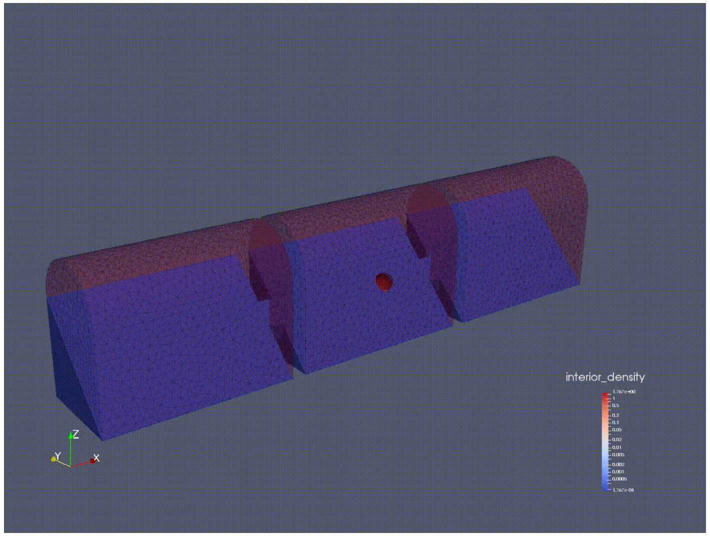
Interconnected environments

PHYSICAL REVIEW E **86**, 046204 (2012)

Impedance and power fluctuations in linear chains of coupled wave chaotic cavities

Gabriele Gradoni,* Thomas M. Antonsen, Jr., and Edward Ott
Institute for Research in Electronics and Applied Physics, University of Maryland, College Park, 20742 Maryland, USA
(Received 7 February 2012; revised manuscript received 16 July 2012; published 5 October 2012)



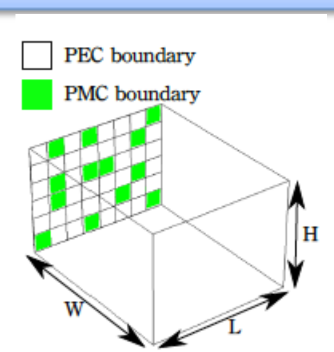
Idea: Propagate Wigner function

$$W(\Omega) = \sum_l T^l W_0(\Omega) \to C_{nm}$$

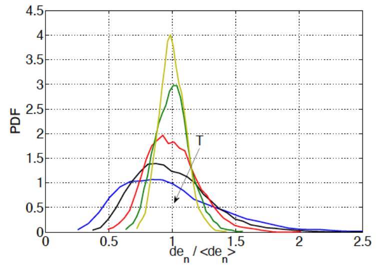
$$I(r) = \int W(r,p)\,dp$$

Ongoing projects

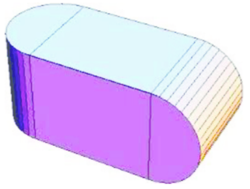
- Controlling wave dynamics, synchronization, *Wave-front/mode shaping, order-from-chaos*



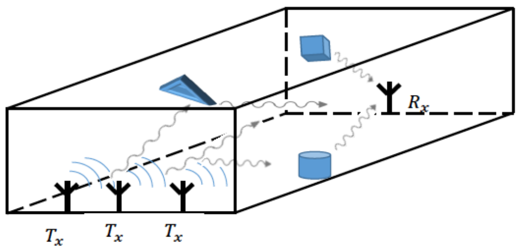
- Transient chaos in semi-open - embedded - systems and scattering media



- Realistic EM environments are neither regular nor irregular: ***Transport of Wigner function for*** *mixed* phase-space



- RCM-based wireless channel transfer matrix for MIMO Systems (**fading statistics from RMT**)



Nottingham Wave Modelling Group

http://wamoresearch.org/

Thank You!

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Papers, research, experimental validations:

http://anlage.umd.edu/RCM/

WAMO Nottingham

Wave Modelling Research Group (*wamoresearch.org*)

George Green.

The University of Nottingham
UNITED KINGDOM · CHINA · MALAYSIA

Mathematics

EEE


Gregor Tanner


Stephen Creagh


Gabriele Gradoni


Steve Greedy


David Thomas

Close Collaboration with

University of Maryland 1856

Institute for Research in Electronics & Applied Physics


Ed Ott


Steven Anlage

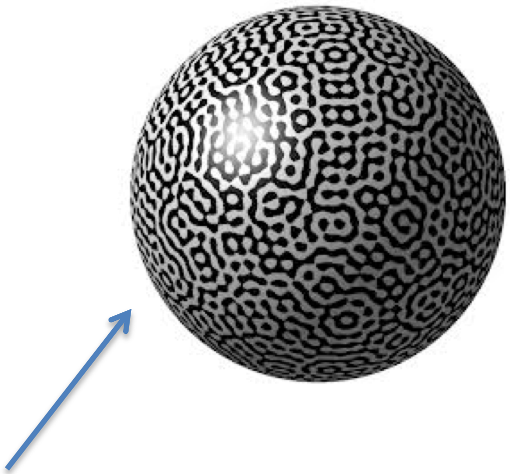

Tom Antonsen

Reverberation Chamber

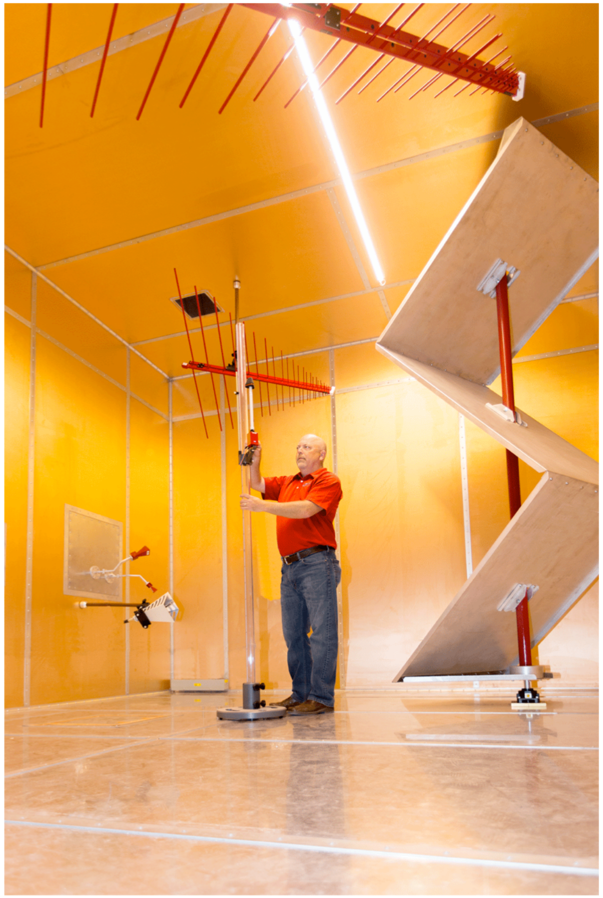
Example of "Complicated" **closed environment**: The mode-stirred RC

Stochastic field generator:

- *Isotropic*
- *Uniform*
- *Ergodic*



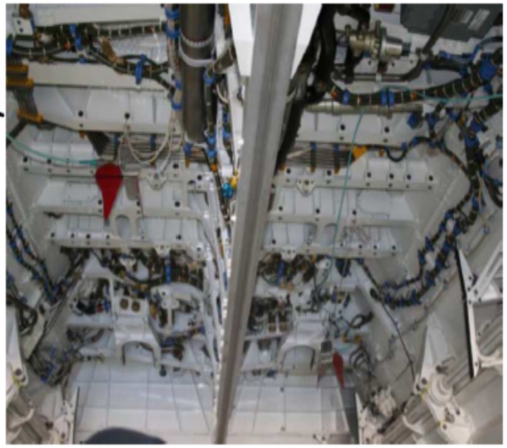
Superposition of multiple reflected waves



Electromagnetic Coupling

Open environments: coupling of external radiation into environment

$\vec{E}, \vec{B}$



Aircraft compartments (Tait, IEEE EMC 2011)

Coupling of external radiation is a complex process (high sensitivity to frequency).

Why?